Prediction of Small Size Pneumatic Tyres for Tyre Traction Properties

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***Abstract* - An analytical approach for determining tyre force and moment characteristics due to pure slips has been presented. The analytical model predicts these forces and moments in terms of easily measurable tyre parameters such as contact patch area, lateral, longitudinal, and vertical stiffnesses of tyre. These analytical predictions show good agreement with existing measured data. The model is suitable for study of vehicle dynamic simulations.**

***Keywords* – slip properties, longitudinal/lateral forces, aligning moment and tyre model.**

I. INTRODUCTION

Tyre traction properties chiefly affect the dynamic behaviour of road vehicles. These properties can be studied either experimentally or analytically. The analytical models have found wide acceptability as they are based on certain tyre parameters which can be measured experimentally in the laboratory with less effort ***more so*** as they are in the equation form. They can be used directly for vehicle dynamic simulations (Dugoff *et al* [1]). Gim *et al* [2] has developed an analytical model for the interaction between a pneumatic tyre and the road surface. The model was based on the assumptions that the contact patch had constant width and the average value of contact pressure over the contact patch width had a parabolic distribution in the circumferential direction. Singh *et al* [3] developed an analytical model for tyre traction properties of small size pneumatic tyres. They assumed uniform pressure distribution along with a constant coefficient of friction between tyre tread and the road surface. Bernard *et al* [4] & Goel *et al* [5], developed an analytical model for tyre traction properties by considering a trapezoidal pressure distribution over the contact patch. In the present study, a simple analytical model for the interaction between pneumatic tyre and the road surface has been developed with the following assumptions.

a) The contact patch is of an elliptical shape.
b) The contact pressure over the contact patch has an inverted boat shape distribution along the circumferential direction.
c) The coefficient of friction between the tyre and road surface is sensitive to both the normal load and the sliding velocity

Due to non-availability of experimental data for small size pneumatic tyres, the results of the analytical model have been verified with the experimental data reported by Sakai [6]. Finally, the analytical model was used to evaluate the tyre traction properties for pure slips of small size pneumatic tyre used in 3-wheeled motor vehicles used in India. Parameters of these tyres have been experimentally measured in the Vehicle Dynamics Laboratory, Indian Institute of Technology, Roorkee (India).

II. TYRE MODEL

A pneumatic tyre-data interaction is generally modelled to have three forces and three moments acting between the tyre and the road surface. Present model considers the tread band connected to the rigid hub by three orthogonal springs whose properties are reported as longitudinal stiffness rate, lateral stiffness rate, and vertical or radial stiffness rate (Bernard *et al* [4]). Longitudinal stiffness rate k_x, and lateral stiffness rate k_y have been defined as the longitudinal force or tread & carcass per unit contact patch area per unit deformation of tread & carcass in longitudinal or lateral direction, respectively. These three springs have interactive functional relationships among each other. It is found from the experiments that the contact patch area between the small size pneumatic tyres and the road surface can be assumed to be elliptical in shape with its major axis in wheel plane as shown in Fig.1 (a). A local orthogonal coordinate system $\xi\eta$ with origin locked at the front extremity of the contact patch is considered such that the ξ -axis coincides with the major axis of the ellipse. The consideration has been done for the analytical purpose. This coordinate system was used as the road co-ordinated system during braking and as the carcass co-ordinated system during traction. The absolute values of the tyre dynamic properties obtained from this co-ordinated system can be transformed to the tyre axis system. Fig. 1(b) shows the pressure distribution P in the foot print area which is considered to be inverted boat shape in the circumferential direction

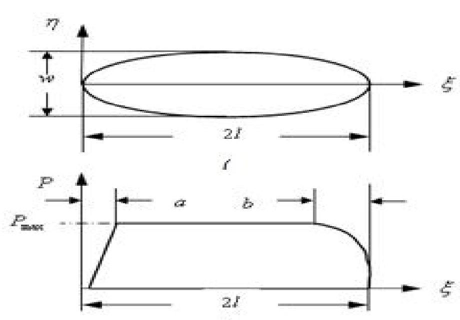

Fig. 1 (a) Tyre contact patch (b) Normal pressure distribution

$$P=\begin{cases} P_{\max}\left[1-\dfrac{(\xi-2l+b)^2}{b^2}\right] & 2l-b\le\xi\le 2l \\ P_{\max} & a\le\xi\le 2l-b \\ P_{\max}\left[\dfrac{\xi}{a}\right] & 0\le\xi\le a \end{cases} \qquad (1)$$

The normal force F_Z was found out by integrating the contact pressure over contact patch area

$$F_Z=\int_0^{2l}\int_{-\eta}^{+\eta} P\,d\xi\,d\eta = 2\int_0^{2l} P\eta\,d\xi = P_{\max}K \qquad (2)$$

Where,

$$K=\frac{C_1^3}{a}\left(\frac{\pi}{2}+\sin^{-1}(A_1)+A_1A_2-\frac{2}{3}A_3\right)$$
$$+w(2l-b-a)-\frac{4C_2^3(b-C_2)}{3b^2}B_3$$
$$+\frac{2C_2^3(2b-C_2)}{b^2}\left(\frac{\pi}{2}-\sin^{-1}(B_1)-B_1B_2\right)- \qquad (3)$$
$$\frac{C_2^4}{b^2}\left(\frac{\pi}{8}-\frac{\sin^{-1}(B_1)}{4}+\frac{\sin(4\sin^{-1}(B_1))}{16}\right)$$

$$C_1=\frac{a}{2}+\frac{w^2}{8a} \quad C_2=\frac{b}{2}+\frac{w^2}{8b}$$

$$A_1=\frac{(a-C_1)}{C_1} \quad A_2=\sqrt{1-\left(\frac{(a-C_1)}{C_1}\right)^2}$$

$$A_3=\left(\sqrt{1-\left(\frac{(a-C_1)}{C_1}\right)^2}\right)^3 \quad B_1=\frac{(C_2-b)}{C_2}$$

$$B_2=\sqrt{1-\left(\frac{(C_2-b)}{C_2}\right)^2} \quad B_3=\left(\sqrt{1-\left(\frac{(C_2-b)}{C_2}\right)^2}\right)^3$$

III. SLIP PROPERTIES

Tyre slip properties are the result of the relative motion between the tyre and the road and prominently influence the tyre dynamic properties. For pure slips (Gim *et al*[2]), the absolute value of longitudinal slip ratio S_x, the lateral slip ratio S_α due to slip angle α and the lateral slip ratio S_γ due to camber angle γ have been given below.

$$S_x=|s| \quad S_\alpha=|\tan\alpha| \qquad S_\gamma=|\sin\gamma| \qquad (4)$$

IV. FRICTION CO-EFFICIENT

The friction coefficient between the tyre and the road surface depends upon several factors. The friction co-efficient μ has been assumed to be a function of the sliding velocity V_s, the normal load F_Z, and the friction coefficient at zero sliding velocity μ_0 (Dugoff *et al*[1])

$$\mu=\mu_0(1-A_sV_s)-\frac{F_LF_Z}{Z} \qquad (5)$$

Where, V_s is a product of forward velocity, V, and the pure slip i.e. or , . , and are load sensitivity function, speed sensitivity function and T&RA load respectively.

V. TRACTION PROPERTIES

Tyre forces and moments have been developed due to the friction coupling between the tyre and road and due to elastic deformations in tyre. In case of pure slips, either longitudinal or lateral deformation was found responsible for the tyre traction properties. The elastic deformation produced elastic stress in the tyre. Under elastic deformation, the contact patch area may be divided into two regions viz the adhesion and sliding. It was assumed that the elastic deformation increased linearly from front extremity of the contact patch toward the breakaway point until the elastic stress reached the frictional stress. It then returned to its undistorted initial position from the breakaway point to the end of the contact patch. The longitudinal elastic stress in adhesion region was expressed as

$$\sigma_\xi^a=k_xS_x\xi \quad \text{for} \quad 0\le\xi\le\xi_S \qquad (6)$$

The longitudinal elastic stress in sliding region which was identical to frictional stress was defined as,

$$\sigma_\xi^s=\mu_xP \qquad \text{For} \qquad \xi_s\le\xi\le 2l \qquad (7)$$

The longitudinal elastic stress in the two regions becomes identical at the breakaway point i.e.

$$\sigma_\xi^a=\sigma_\xi^s \qquad \text{at } \xi=\xi_s \qquad (8)$$

Equation (8**)** provided an expression for the limits of slip ratio for the elastic deformation and the stress. Upper limit gave the critical value of the slip ratio above which the deformation reached its maximum value and sliding is experienced in the full contact patch area. Two cases can be considered for the analytical evaluation of traction properties. The first case was for the elastic deformation and the second case was for the sliding without any elastic deformation.

A. *Case-I: For Elastic Deformation*

The longitudinal force was obtained by integrating the longitudinal stress over the contact patch area

$$F_\xi=\int_0^{\xi s}\int_{-\eta}^{+\eta}\sigma_\xi^a\,d\xi\,d\eta+\int_{\xi s}^{2l}\int_{-\eta}^{+\eta}\sigma_\xi^s\,d\xi\,d\eta \qquad (9)$$

B. Case-II: For Sliding without any Elastic Deformation

In case of the pure sliding, there existed no elastic deformation and elastic stress, only a frictional stress was found to occur. Thus, the longitudinal force was found to depend only on a frictional force. i.e.

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$$F_{\xi} = \mu_x F_Z \tag{10}$$

In similar way, analytical expressions for other traction properties due to pure slip angle and due to pure camber angle have been evaluated. The various tyre parameters have been measured in the laboratory. These experimentally obtained tyre parameters for MRF 4.00-8, 6 Bias ply tyre at inflation pressure of 207 kPa have been presented in Table 1[5].

TABLE I
EXPERIMENTAL TYRE PARAMETERS

Load (N)	1540	2000	2465
$2l$(m)	0.104	0.116	0.126
w(m)	0.076	0.08	0.082
b(m)	0.023	0.021	0.023
k_x(kN/m^3)	19303	16568	15807
k_y(kN/m^3)	10211	8321	7229
k_z(kN/m^3)	151	158	169
R_e(m)	0.2	0.197	0.195

VI. RESULTS

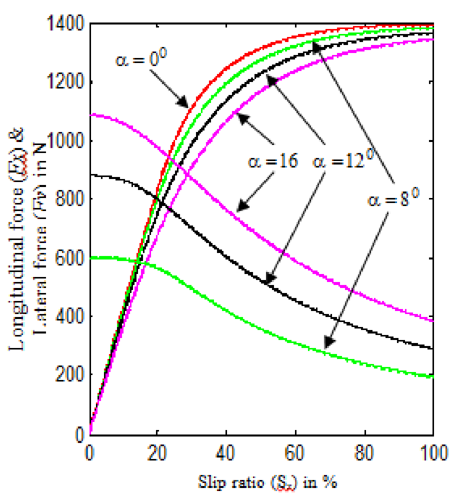

Fig. 2. Longitudinal force & Lateral force due to slip ratio

It is observed (Fig. 2) that the longitudinal force has increased almost linearly up to a certain value of slip ratio. As the absolute value of slip ratio has increased, it continuous to increase non-linearly until it reached a maximum value.

Similar to the longitudinal force, the lateral force due to pure slip angle shows a tendency of linear increase as the absolute value of this angle had increased from zero. Thereafter, the lateral force has increased non-linearly to its maximum value, than it has decreased either linearly or non-linearly, depending on the friction force as shown in Fig.3. Due to very limited availability of experimental data regarding tyre traction properties for small size pneumatic tyres, the experimental data reported by Sakai [6] has been used for the validation of the analytical model. It was observed that the analytical values show good agreement with the experimental values for both longitudinal and lateral forces. The inverted boat shape pressure distribution over the contact patch area in the analytical model is dependent upon the value of $a/2l$ & $b/2l$. The values of $a/2l$ & $b/2l$, which are the ratios of the contact patch length to the total contact patch length, where the pressure was found to have linearly increased or decreased. These values have been chosen to vary from 0.18 to 0.23 corresponding to the tyre normal load ranging from 1540 N to 2465 N. The tyre traction properties which have been obtained from the analytical model have been generally found to show good agreement with the experimental data reported by Sakai [6] except in the case of the aligning torque.

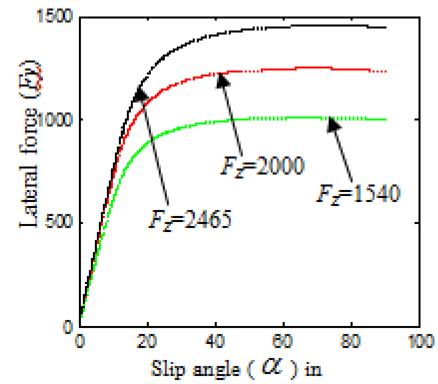

Fig. 3. Lateral force due to slip angle for different loads

The difference in the tyre data w.r.t to the aligning torque has also been reported by Gim *et al*[2]. However the difference in the aligning torque value has been found to be very small when compared to the values reported by Gim *et al* [2]. The model is well suited for dynamic simulation of the vehicle. The tyre traction properties of MRF 4.00-8, 6 Bias ply tyre was predicted analytically by taking the value of $b/2l$ obtained experimentally at different normal loads.

Due to lack of adequate experimental set-up to determine the coefficient of friction between the tyre tread and the road surface for above tyre, the data taken by Gim *et al* [2] has been considered for analysis as the tyre size was comparable with the size of above tyre.

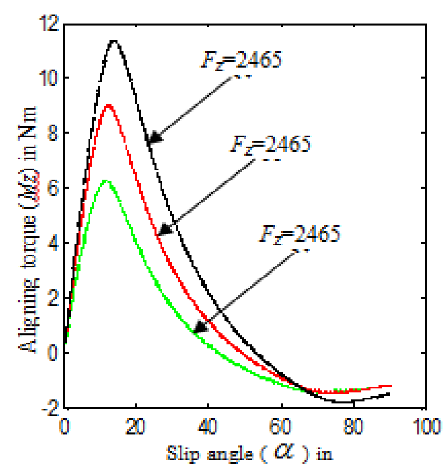

Fig. 4. Aligning torque vs slip angle for different loads

VII. CONCLUSIONS

The present analytical model was based on experimentally obtained tyre parameters. The model is simple and easy to incorporate in the vehicle dynamic simulations. The inverted boat shape pressure distribution proved to be more realistic as compared to other pressure distributions since at the upper limit of slip property, the sliding without any elastic deformation has taken place at $\xi_s = a$ and not at the front extremity of the contact patch area $\xi_s = 0$. The values of $b/2l$ have been obtained from the actual tyre measurements, which make the model a better representative of the tyre behaviour. The difference in the aligning obtained from this analytical model with the data reported by Sakai [6] would have been due to consideration of the pressure distribution in the transverse direction in the contact patch as uniform. There is a further possibility of extending this analysis by including the pressure distribution variation along the width of the contact patch and/or by considering an asymmetric pressure distribution over the contact patch area.

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